

Mixed Quark-Gluon condensate at finite temperature and density in the global color symmetry model

Zhao Zhang¹, Wei-qin Zhao^{1,2}

¹*Institute of High Energy Physics, Academia Sinica,
Beijing, 100039, P. R. China*

²*CCAST(World Laboratory), P.O. Box 8730,
Beijing 100080, P. R. China*

Abstract

The mixed quark-gluon condensate is another chiral order parameter in QCD, which plays an important role in the application of QCD sum rules. In this letter, we study the properties of quark-gluon mixed condensate at finite temperature and quark chemical potential in the framework of global color symmetry model. Using an infrared-dominant model gluon two-point function, we find that the behavior of quark-gluon mixed condensate at finite temperature and chemical potential is similar to that of the quark condensate, and both of them give the same information about chiral phase transition. We also find that the ratio of these two condensates is insensitive to the temperature and chemical potential, which supports the conclusion obtained recently by the authors using quenched lattice QCD (They only studied the nature of the mixed quark-gluon condensate at finite temperature).

Keywords: Field theory at finite temperature and chemical potential; Vacuum condensate; Confinement; Dynamical chiral symmetry breaking; Dyson-Schwinger equations.

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⁰E-mail:zhangforever@sohu.com

The mixed quark-gluon condensates $g\langle\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\rangle$ is one of the vacuum expectation values which reflect the nonperturbative structure of QCD vacuum. This condensate plays an important role for the low energy phenomena of hadrons in the framework of QCD sum rules[1, 2], where the various condensates in the operator product expansion (OPE) are taken as input parameters phenomenologically to reproduce various hadronic properties systematically. Contrary to the well-known condensates of the lowest dimension, $\langle G^2_{\mu\nu}\rangle$ and $\langle\bar{q}q\rangle$, the mixed condensate represents the direct correlation between quarks and gluons in QCD vacuum. Due to the fact that the mixed condensate plays a relevant role in OPE as a next-to-leading chiral variant operator, the mixed quark-gluon condensate can be used as another chiral order parameter of the second lowest dimension for QCD phase transition.

The previous evaluations of the mixed quark-gluon condensate at zero temperature and chemical potential were performed in the framework of QCD sum rule method[2, 3, 4], quenched lattice QCD[5, 6] and QCD effective models[7, 8], respectively. The value of this condensate is still not conclusive from these studies, and the parameter $m_0^2 = g\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle/\langle\bar{q}q\rangle$ determined at the renormalization point $u = 1\text{GeV}^2$ ranges from 0.5GeV^2 to 3.7GeV^2 . Recently, the behaviour of the various low-dimensional condensates at finite temperature and chemical potential is the subject of intensive researches due to their direct relation to the QCD phase transition. In contrast to $\langle\bar{q}q\rangle$, the thermal properties of the mixed quark-gluon condensate of dimension 5 is still least known. Therefore it is interesting to investigate the thermal nature of this mixed condensate for it can act as the chiral order parameter which is independent of the quark condensation $\langle\bar{q}q\rangle$. Furthermore, the evaluation of the mixed quark-gluon condensate at finite temperature T and quark chemical potential μ has an impact on the hadron phenomenology through QCD sum rule.

So far, the investigation of thermal behavior of $g\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle$ has been performed by the authors[5] through the quenched lattice QCD calculation with Kogut-Susskind fermion(KS) field method, where a critical temperature $T_c \approx 280\text{MeV}$ was obtained for both the chiral symmetry and deconfinement restoration transitions. In Ref.[5], the obtained ratio between the mixed quark-gluon condensate and the quark condensate is almost independent of the temperature even in the very vicinity of T_c . Since there is sig-

nificant overlap between contemporary Dyson-Schwinger-Equations (DSEs) studies and the numerical simulation of lattice QCD, the DSEs could provide an adjunct to lattice QCD. The truncation that is accurate in the common domain such as zero temperature and chemical potential and finite temperature can be used to extrapolate into the finite chemical potential domain, which is presently inaccessible in lattice simulations. Therefore, it is necessary to elucidate the (T, μ) dependence of the mixed quark-gluon condensate through DSE-models. In this letter, we will explore the properties of the mixed condensate at finite temperature and chemical potential in the framework of the global color symmetry model(GCM).

As a truncated DSE-model, GCM is a quite successful four-fermion interaction field theory which can be directly derived through a truncation of QCD[10]. This truncated DSE model has been applied extensively at zero temperature and chemical potential to the phenomenology of QCD[11, 12], including the studies of observables from strong interaction to weak interaction area. Furthermore, the truncated DSE models also have made important progress in the studies of strong QCD at finite temperature and chemical potential[13].

In a Euclidean space formulation, with $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ and $\gamma_\mu^+ = \gamma_\mu$, the inverse of dressed-quark propagator at finite temperature and chemical potential in the framework of GCM takes the form

$$S^{-1}(\tilde{p}_k) = i\vec{\gamma} \cdot \vec{p} + i\gamma_4(\omega_k + i\mu) + \Sigma(\tilde{p}_k) \quad (1)$$

$$= i\vec{\gamma} \cdot \vec{p} A(\tilde{p}_k) + i\gamma_4(\omega_k + i\mu) C(\tilde{p}_k) + B(\tilde{p}_k), \quad (2)$$

where $\tilde{p}_k \equiv (\vec{p}, \tilde{\omega}_k)$, $\tilde{\omega}_k = \omega_k + i\mu$, $\omega_k = (2k+1)\pi T$ is the Fermi Matsubara frequency, μ is the chemical potential and $\Sigma(\tilde{p}_k)$ is the self-energy term of the dressed quark propagator. The gap equation for the dressed quark in the chiral limit is determined by the rainbow DSE

$$(A(\tilde{p}_k) - 1)\tilde{p}^2 = \frac{8}{3} \int_{l,q} g^2 D(\tilde{p}_k - \tilde{q}_l) \frac{A(\tilde{q}_l) \vec{q} \cdot \vec{p}}{A^2(\tilde{q}_l) \vec{q}^2 + C^2(\tilde{q}_l) \tilde{\omega}_l^2 + B^2(\tilde{q}_l)} \quad (3)$$

$$(C(\tilde{p}_k) - 1)\tilde{\omega}_k^2 = \frac{8}{3} \int_{l,q} g^2 D(\tilde{p}_k - \tilde{q}_l) \frac{C(\tilde{q}_l) \tilde{p}_k \cdot \tilde{q}_l}{A^2(\tilde{q}_l) \vec{q}^2 + C^2(\tilde{q}_l) \tilde{\omega}_l^2 + B^2(\tilde{q}_l)} \quad (4)$$

$$B(\tilde{p}_k) = \frac{16}{3} \int_{l,q} g^2 D(\tilde{p}_k - \tilde{q}_l) \frac{B(\tilde{q}_l)}{A^2(\tilde{q}_l) \vec{q}^2 + C^2(\tilde{q}_l) \tilde{\omega}_l^2 + B^2(\tilde{q}_l)}, \quad (5)$$

where $\int_{l,q} = \beta^{-1} \sum_{l=-\infty}^{\infty} \int \frac{d^3 q}{(2\pi)^3}$ and $\beta^{-1} = T$. The term $g^2 D(\tilde{p}_k)$ is the effective dressed-gluon propagator at finite temperature and chemical po-

tential, which is treated as an input parameter in the quark gap equation. The complex scalar functions $A(\tilde{p}_k)$, $B(\tilde{p}_k)$ and $C(\tilde{p}_k)$ satisfy: $F(\tilde{p}_k)^* = F(\tilde{p}_{-(k+1)})$, $F = A, B, C$, although not explicitly indicated they are functions only of \tilde{q}^2 and $\tilde{\omega}_k^2$.

We will use the technique proposed in Ref.[7] to calculate the mixed quark-gluon condensate at finite temperature and chemical potential in the framework of global color symmetry model. In the context of nonzero temperature, the space-time integration $\int d^4x$ is replaced by the form $\int_0^\beta dx_4 \int d^3x$ and the continuum integration $\int d^4p$ in the momentum space is replaced by Matasubara frequency summation $\beta^{-1} \sum_{k=-\infty}^{\infty} \int d^3p$. At the mean field level, the vacuum expectation of any quark operator of the form

$$O_n \equiv (\bar{q}_{j_1} \Lambda_{j_1 i_1}^{(1)} q_{i_1}) (\bar{q}_{j_2} \Lambda_{j_2 i_2}^{(2)} q_{i_2}) \cdots (\bar{q}_{j_n} \Lambda_{j_n i_n}^{(n)} q_{i_n}) \quad (6)$$

can be calculated straightforward. Here $\Lambda^{(i)}$ is an operator in Dirac, flavor and color space. Since the generating functional for the quark field is a typical Gaussian type integration at the mean field level in the framework of this four-fermion interaction model, the expression for the vacuum expectation of quark operator O_n can be easily derived[18, 7] and takes the form

$$\langle : O_n : \rangle = (-1)^n \sum_p (-)^p [\Lambda_{j_1 i_1}^{(1)} \cdots \Lambda_{j_n i_n}^{(n)} (S)_{i_1 j_{p(1)}} \cdots (S)_{i_n j_{p(n)}}], \quad (7)$$

where Π stands for a permutation of the n indices and S is the dressed-quark propagator in the mean field level. For example, we can get the familiar expression for the quark condensate $\langle \bar{q}q \rangle_T^\mu$ at temperature T and chemical potential μ

$$\begin{aligned} \langle \bar{q}q \rangle_T^\mu &= (-) \text{Tr}_{DC} S(x, x) \\ &= (-) 4N_c \int_{l,q} \frac{B(\tilde{q}_l)}{A^2(\tilde{q}_l) \tilde{q}^2 + C^2(\tilde{q}_l) \tilde{\omega}_l^2 + B^2(\tilde{q}_l)}. \end{aligned} \quad (8)$$

The trace is to be taken in Dirac and color space, whereas the flavor trace has been separated out. The four quark condensate can also be obtained easily from Eq.(7) and proved to be proportional to the square of the quark condensate which is consistent with the vacuum saturation assumption of Ref.[7].

Due to the glounic degree in $g\langle \bar{q}\sigma_{\mu\nu} G^{\mu\nu} q \rangle$, we can not get the expression for the mixed quark-gluon condensation directly from Eq.(7). Since the generating functional over gluon field A in this four-fermion interaction model is

quadratic with a given quark-quark interaction D , the integration over any number of gluon fields can be performed analytically [7]. Using the same shorthand notation for the typical Gaussian integrations as in Ref. [7], we have

$$\int \mathcal{D}A e^{-\frac{1}{2}AD^{-1}A+jA} \equiv e^{\frac{1}{2}jDj} \quad (9)$$

$$\int \mathcal{D}AA e^{-\frac{1}{2}AD^{-1}A+jA} \equiv (jD)e^{\frac{1}{2}jDj} \quad (10)$$

$$\int \mathcal{D}AA^2 e^{-\frac{1}{2}AD^{-1}A+jA} \equiv [D + (jD)^2]e^{\frac{1}{2}jDj}, \quad (11)$$

where $j = \bar{q}\gamma_\mu \frac{\lambda^a}{2}q$ is the quark color current. It should be noted that D stands for the connected gluon two-point Green function at nonzero temperature and chemical potential and the integration in the exponent takes the form $\int_0^\beta dx_4 \int d^3x$ according to the imaginary time thermal field theory. Due to the transition from the gluon vacuum average to the quark color current $\bar{q}\gamma_\mu \frac{\lambda^a}{2}q$ together with the gluon two-point function D which is usually treated as an input parameter in truncated DSE-model method to get the quark gap equation, we can use the Eq.(7) to evaluate the mixed quark-gluon condensate at temperature T and chemical potential μ in the mean field level. Applying the technique described above, we obtain

$$\begin{aligned} g\langle \bar{q}(x)G_{\mu\nu}(x)\sigma^{\mu\nu}q(x) \rangle &= (-2i)N_c \frac{4}{3} \int_z [\partial_\mu^{(x)} g^2 D(z-x)] \\ &\quad \times \text{tr}_D [S(z-x)\sigma_{\mu\nu}S(x-z)\gamma_\nu] \\ &\quad + (4i)N_c \int_{z_1} \int_{z_2} g^2 D(z_1-x)g^2 D(z_2-x) \\ &\quad \times \text{tr}_D [S(z_2-x)\sigma_{\mu\nu}S(x-z_1)\gamma_\mu S(z_1-z_2)\gamma_\nu], \end{aligned} \quad (12)$$

where $x = (x_4, \vec{x})$ and \int_x stands for $\int_0^\beta dx_4 \int d^3x$.

From the dressed quark gap equation in GCM

$$\Sigma(x-y) = \frac{4}{3}g^2 D(x-y)\gamma_\mu S(x-y)\gamma_\mu, \quad (13)$$

we can get the form

$$\frac{4}{3}g^2 D(x-y)S(x-y) = \frac{1}{4}\frac{1}{4}\text{tr}_D [\Sigma(x-y)] - \frac{1}{2}\frac{1}{4}\gamma_\mu \text{tr}_D [\Sigma(x-y)\gamma_\mu]. \quad (14)$$

Using this equation, the integration over the gluon two-point function D in Eq.(12) is replaced by the quark self-energy Σ . This technique strongly

simplifies the calculation of the mixed condensate. The formula for the calculation of the mixed quark-gluon condensate is now expressed only in terms of three scalar functions A, B and C which are solutions of the quark gap equation. After Fourier transformation and performing the integration over space-time, we get the final integration expression for the mixed condensate at temperature T and chemical potential μ .

$$g\langle\bar{q}G_{\mu\nu}\sigma^{\mu\nu}q\rangle_T^\mu = 36\int_{k,q}\frac{B[(2-A)\bar{q}^2+(2-C)\tilde{\omega}_k^2]}{A^2\bar{q}^2+C^2\tilde{\omega}_k^2+B^2} + \frac{81}{4}\int_{k,q}\frac{B^3+2B[A(A-1)\bar{q}^2+C(C-1)\tilde{\omega}_k^2]}{A^2\bar{q}^2+C^2\tilde{\omega}_k^2+B^2}. \quad (15)$$

Substituting $T = 0$ and $\mu = 0$ into Eq.(15), We have

$$g\langle\bar{q}G_{\mu\nu}\sigma^{\mu\nu}q\rangle_{T=0}^{\mu=0} = (-)\frac{3}{16\pi^2}\left\{12\int ds s^2 B\frac{2-A}{sA^2+B^2} + \frac{27}{4}\int ds s B\frac{[2A(A-1)]s+B^2}{sA^2+B^2}\right\}. \quad (16)$$

This is just the equation obtained in Ref.[7]. It is clear according to Eq.(15) that in the Wigner phase characterized by $B \equiv 0$, the mixed quark-gluon condensate takes the value 0 and the nonzero value of this condensate signals the nontrivial structure of QCD vacuum. This is the reason that $g\langle\bar{q}G_{\mu\nu}\sigma^{\mu\nu}q\rangle$ can also be used as the chiral order parameter as $\langle\bar{q}q\rangle$ does.

It should be noted that our study ignores effects from hard gluonic radiative corrections to these condensates due to its minor importance for our study of nonperturbative properties in the low and medium energy region. That means the chosen quark-quark interaction model g^2D has a finite range in momentum space and the momentum integrals in Eq.(15) are finite. Because GCM is a no-renormalizable effective field theory, the scale at which a condensate is defined in our approach is a typical hadronic scale, which is implicitly determined by the model gluon propagator g^2D and the solutions of DSE for the dressed quark gap equations. The similar situation is the determination of vacuum condensates within the framework of the instanton liquid model[21] where the scale is set by the inverse instanton size.

To explore the properties of the mixed quark-gluon condensate at finite temperature and chemical potential, we use the infrared-dominant model

$$g^2D(\tilde{p}_k) = \frac{3}{16}(2\pi)^3\beta\eta^2\delta_{k0}\delta(\vec{p}), \quad (17)$$

which is a generalization to $T \neq 0$ of the model introduced in Ref.[19], where $\eta \approx 1.06\text{GeV}$ is a mass-scale parameter fixed by fitting π - and ρ -meson masses. As an infrared enhanced model, it does not represent well the behavior of $D_{\mu\nu}(\tilde{p}_k)$ away from $|\vec{p}|^2 + \tilde{\omega}_k^2 \approx 0$. Consequently some model-dependent artifacts arise. However and there is significant advantage in its simplicity and since the artifacts are easily identified, the model can exhibits many of the qualitative features based on the more sophisticated *Ansätze*[15, 16]. This simple confining model has been used successfully to explore the thermal properties of QCD[14].

Substituting Eq.(17) into Eq.(15), we get an algebraic equation which has two qualitatively distinct solutions. The Nambu-Goldstone solution, with

$$A(\tilde{p}_k) = C(\tilde{p}_k) = \begin{cases} 2, & \text{Re}(\tilde{p}_k^2) < \frac{\eta^2}{4}, \\ \frac{1}{2}(1 + \sqrt{1 + \frac{2\eta^2}{\tilde{p}_k^2}}), & \text{otherwise,} \end{cases} \quad (18)$$

$$B(\tilde{p}_k) = \begin{cases} \sqrt{\eta^2 - 4\tilde{p}_k^2}, & \text{Re}(\tilde{p}_k^2) < \frac{\eta^2}{4}, \\ 0, & \text{otherwise,} \end{cases} \quad (19)$$

describes a phase of the model in which: 1) chiral symmetry is dynamically broken due to the nonzero quark mass function, $B(\tilde{p}_k)$, in the chiral limit; and 2) the dressed-quarks are confined due to the absence of a Lehmann representation of the dressed quark propagator. The other one is Wigner solution with for which

$$\hat{B}(\tilde{p}_k) \equiv 0, \quad \hat{A}(\tilde{p}_k) = \hat{C}(\tilde{p}_k) = \frac{1}{2}(1 + \sqrt{1 + \frac{2\eta^2}{\tilde{p}_k^2}}), \quad (20)$$

which describes a phase of the model with neither dynamical chiral symmetry breaking(DCSB) nor confinement.

In Ref.[14], this model was used to explore the chiral symmetry and deconfinement restoration transition between these two phases characterized by qualitatively different , momentum-dependent modification of the quark propagator. The relative stability of the confined and deconfined phases is measured by the (T, μ) -dependent vacuum pressure difference or bag constant[10]

$$B(T, \mu) \equiv P[S_{NG}] - P[S_W], \quad (21)$$

where S_{NG} stands for the quark propagator with the Nambu-Goldstone solution and S_W means the quark propagator with the Wigner solution.

$P[S]$ is the pressure obtained by the tree-level auxiliary-field effective action in the stationary phase approximation[17], which takes the form

$$P[S] = \frac{T}{V} \{Tr Ln[\beta S^{-1}] - \frac{1}{2}Tr[\Sigma S]\} \quad (22)$$

$B(T, \mu) > 0$ indicates the stability of the confined phase and hence the phase boundary is specified by

$$B(T, \mu) = 0. \quad (23)$$

In the chiral limit, the deconfinement and chiral symmetry restoration transition in this model are coincident. Substituting Eq.(18-20) into Eq.(23), we get the equation[14]

$$\frac{2N_c N_f}{\pi^2} \beta^{-1} \sum_{l=0}^{l_{max}} \int_0^{\bar{p}} dp p^2 [Re(\frac{2\tilde{p}_k^2}{\eta^2} - \hat{C}(\tilde{p}_k)^{-1}) - \ln |\frac{\tilde{p}_k^2 \hat{C}(\tilde{p}_k)^2}{\eta^2}|] = 0 \quad (24)$$

with $\omega_{l_{max}}^2 \leq \frac{\eta^2}{4} + \mu^2$, $\bar{p}^2 = \omega_{l_{max}}^2 - \omega_l^2$, which determines the critical line in the (T, μ) plane. The critical temperature for zero chemical potential in this model is $T_c^0 = 0.170\text{Gev}$ which is only 12% larger than the value obtained in Ref.[15] using a more sophisticated model and the order of transition is the same. This illustrates the simple model's ability to provide a reasonable guide to the thermodynamics properties of more sophisticated DSE-models. For $T = 0$, the critical chemical potential is $\mu_c^0 = 0.3\text{GeV}$ and in case of nonzero chemical potential the transition is the first order[14, 16].

Substituting Eqs.(18,19) into Eq.(8) and Eq.(15), we have the expressions for quark condensate and mixed quark-gluon condensate at temperature T and chemical potential μ from the infrared-dominant model

$$-\langle \bar{q}q \rangle = \frac{12}{\pi^2} \frac{T}{\eta^2} \sum_{l=0}^{l_{max}} \int_0^{\bar{p}} dp p^2 Re(\sqrt{\eta^2 - 4(p^2 - \mu^2 + \omega_l^2 + 2i\mu\omega_l)}) \quad (25)$$

$$-g\langle \bar{q}\sigma Gq \rangle = \frac{81}{4\pi^2} T \sum_{l=0}^{l_{max}} \int_0^{\bar{p}} dp p^2 Re(\sqrt{\eta^2 - 4(p^2 - \mu^2 + \omega_l^2 + 2i\mu\omega_l)}). \quad (26)$$

Comparing Eq.(26) with Eq.(25), we find that the mixed quark-gluon condensate is proportional to the quark condensate and the ratio between these two condensates is a constant independent of temperature T and chemical potential μ . It is a nontrivial result that the behavior of the mixed quark-gluon condensate at finite temperature and chemical potential is similar to the quark condensate, for $g\langle \bar{q}\sigma Gq \rangle$ reflects the color-octet component

of quark-antiquark pairs in the QCD vacuum, while $\langle \bar{q}q \rangle$ reflects only the color-singlet quark-antiquark components. That means the mixed condensate $g\langle \bar{q}\sigma Gq \rangle$ represents the direct correlation between color-octet $q-\bar{q}$ pairs and the gluon field strength $G_{\mu\nu}$, while $\langle \bar{q}q \rangle$ represents the direct correlation between quarks and antiquarks. This nontrivial result was also obtained in Ref.[9] by quenched lattice QCD simulation with the KS-fermion method (note that in Ref.[9], the authors only explored the temperature dependence of the mixed condensate and the critical temperature for chiral symmetry restoration is 280MeV). The ratio $m_0^2 = g\langle \bar{q}\sigma Gq \rangle / \langle \bar{q}q \rangle$ is 1.90GeV^2 in this infrared-dominant model, which suggests the significance of the mixed condensate in OPE.

In Ref.[20], these two condensates take the form

$$\langle \bar{q}q \rangle = \frac{1}{V} \int d\lambda \frac{m\rho(\lambda)}{\lambda^2 + m^2}, \quad (27)$$

$$g\langle \bar{q}\sigma Gq \rangle = \frac{1}{V} \int d\lambda \frac{m\rho(\lambda)}{\lambda^2 + m^2} \langle \lambda | \sigma G | \lambda \rangle, \quad (28)$$

where $|\lambda\rangle$ stands for the eigenvector of the Dirac operator as $i\hat{D}|\lambda\rangle = \lambda|\lambda\rangle$, and $\rho(\lambda)$ the spectral density on λ . So the ratio m_0^2 can be expressed as $\langle \lambda | \sigma G | \lambda \rangle_{\lambda=0}$ in the chiral limit. The insensitive (T, μ) dependence of m_0^2 suggests that the value of $\langle \lambda | \sigma G | \lambda \rangle_{\lambda=0}$ has a weak dependence on both the temperature and chemical potential in the Nambu-Goldstone phase, even in the very vicinity of critical point.

In Fig.1, the thermal effect on $-g\langle \bar{q}\sigma Gq \rangle$ is plotted against temperature T at several chosen chemical potential points. It is clearly indicated from Fig.1 that $-g\langle \bar{q}\sigma Gq \rangle$ decreases continuously to zero with increasing T . The chemical potential dependence of $-g\langle \bar{q}\sigma Gq \rangle$ is shown in Fig.2 and obviously the mixed condensate increases with μ , up to a critical value $\mu_c(T)$ when it drops discontinuously to zero. This behavior is the same as $-\langle \bar{q}q \rangle$, which can be attributed to the combination $\mu^2 - \omega_l^2$ appearing in Eq.(25) and Eq.(26).

We can see from above analysis that though $g\langle \bar{q}\sigma Gq \rangle$ and $\langle \bar{q}q \rangle$ characterize different aspects of QCD vacuum, both of them not only give the same critical point for the chiral symmetry restoration, but also have the same critical behavior for the QCD phase transition. This conclusion confirms the result in Ref.[9] and seems to indicate that all order parameters for a phase transition have the universal critical behavior near the critical point.

In summary, we have estimated the (T, μ) dependence of $g\langle \bar{q}\sigma Gq \rangle$ in the

framework of GCM. Using a simple, confined DSE-model of QCD, we get that the value of the mixed quark-gluon condensate is proportional to the quark condensate and the ratio between these two condensates is independent of T and μ . The large ratio $m_o^2 = 1.90\text{GeV}^2$ suggests that the mixed quark-gluon condensate plays an important role in QCD sum rules. As two low-dimensional chiral order parameters, the mixed quark-gluon condensate and quark condensate give the same critical behavior for chiral symmetry restoration. As mentioned above, this infrared-dominant model is poor to represent the ultraviolet behavior of the gluon propagator, so there may exist some model-dependent artifacts in our study. Because this model can provide a reasonable guide to the thermodynamics properties based on more sophisticated DSE-models of QCD, we believe that these results are qualitatively coincident with the more complex *Ansätze*. The more sophisticated investigation is under progress.

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Figures

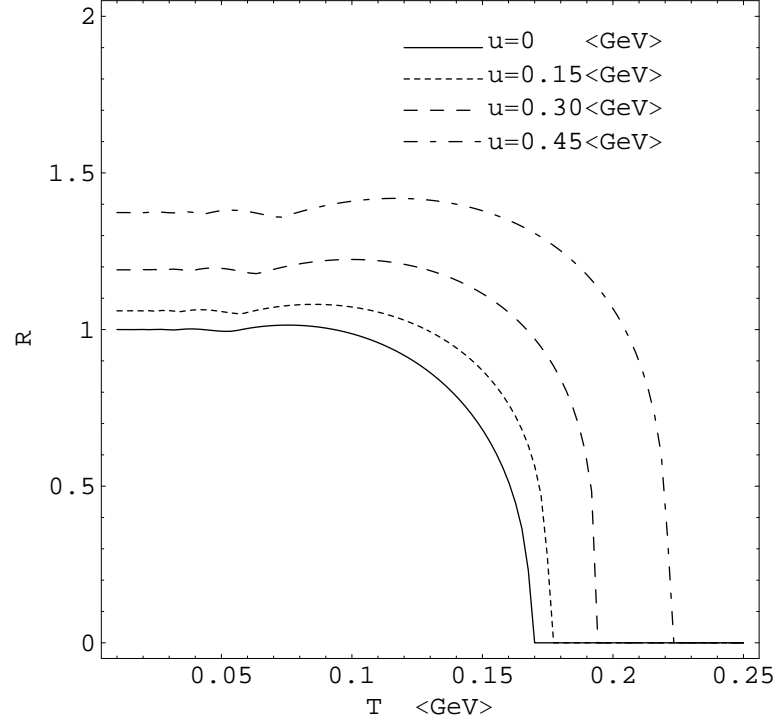


Fig.1. The ratio $R = (g\langle\bar{q}\sigma Gq\rangle_T^\mu)^{(\frac{1}{5})}/(g\langle\bar{q}\sigma Gq\rangle_{T=0}^{\mu=0})^{(\frac{1}{5})}$, as a function of T for a range values of μ . The “structure” in those curves, apparent for small T , is an artifact of the fact that the infrared-dominant model does not represent well the the quark-quark interaction in the ultraviolet domain[14].

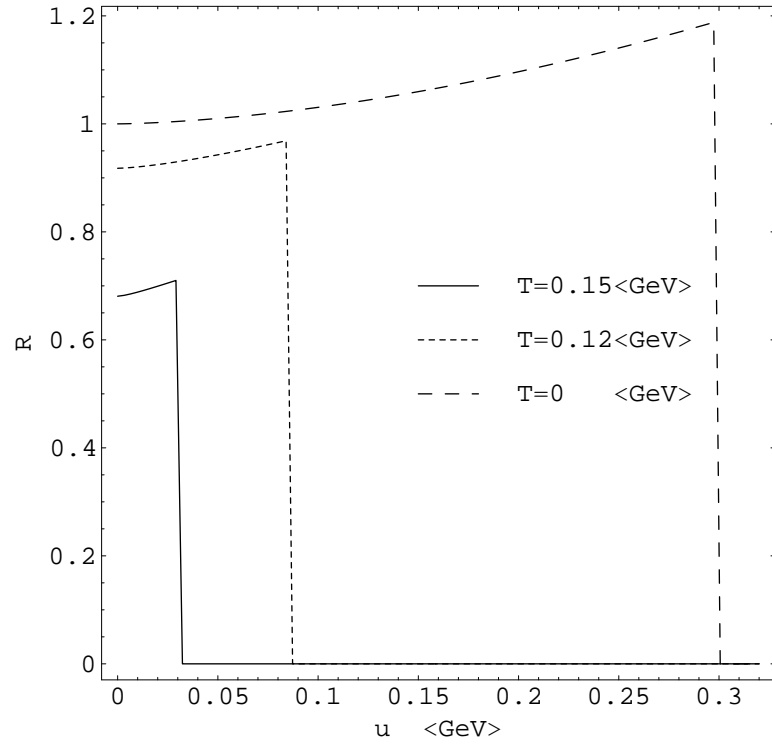


Fig.2.The ratio $R = (g\langle\bar{q}\sigma Gq\rangle_T^\mu)^{(\frac{1}{5})}/(g\langle\bar{q}\sigma Gq\rangle_{T=0}^{\mu=0})^{(\frac{1}{5})}$, as a function of μ for a range values of T .